THERMODIFFUSION IN VISCOELASTIC SOLIDS

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Streszczenie: Przedstawiono trzy równoważne sformułowania zagadnień termodyfuzji w ciele lepkosprężystym. Rozważania przeprowadzono postulując postać bilansów masy, pędu, krętu, energii oraz entropii dla migracji masy i ciepła w ciele stałym. Następnie otrzymano równania fizyczne procesu po przyjęciu postaci trzech równoważnych potencjałów termodynamicznych.

Abstract: Three equivalent versions of the problems of formulation of thermodiffusion in viscoelastic solids are presented. The balances of mass, momentum, angular momentum, energy and entropy for migration of mass and heat in a solid are proposed. Next the physical equations of the process for three equivalent thermodynamic potentials are obtained. Physical equations of the process have been obtained assuming three equivalent forms of thermodynamic potentials.

Резюме: В работе представлены три эквивалентные формулировки вопросов термодиффузии в вязкоупругом теле. Провели рассуждения, предлагая вид балансов массы, количества движения, кинематического момента, энергии и энтропи для переноса массы и тепла в твердом теле. Затем получили физические уравнения процесса после принятия трех эквивалентных термодинамических потенциалов.

Résumé: On a présenté trois énoncés équivalents des problèmes de la thermodiffusion dans un corps viscoélastique. Les examens ont été faits en supposant la forme des bilans de la masse, de la vitesse acquise, du moment cinétique, de l'énergie et de l'entropie pour la migration de la masse et de la chaleur dans un corps solide. On a obtenu l'équation physique du processus après avoir admis trois potentiels thermodynamiques équivalents.

1. INTRODUCTION

Mass diffusion in a solid is a physical process often applied in new technologies such as diffusive binding of heterogeneous materials, accelerated heat treatment of fresh concrete, protective coating of metals (for instance aluminized coatings). Flows of a diffusive character occur also in many corrosive processes (see [18]). Diffusion processes belong to the physical phenomena which are usually coupled with the effects of other fields, of which heat flows, electric flows or stress states are the most frequent. The most important effect is due to heat flows which activate mass migration. Coupling of heat and diffusive flows leads to thermodiffusion. Experimental results concerning heat treatment of curing concrete [15] are a good example of such couplings, although heat exchange and moisture exchange in curing concrete are much more complex because of the porosity and not comple-

ted hydration of concrete. Pressing of fresh concrete used for acceleration of its setting and curing proves that the stress field is also involved. Physically similar phenomena take place during diffusive binding of different materials, e.g., metals with ceramics. The processes proceed at elevated temperature, often in vacuum and under the pressure exerted on the elements being joined. In view of the facts mentioned above, the formulation of a general theory in which interaction of diffusion field, stress field and heat flow field should be taken into consideration seems necessary. Thermodiffusion is just the theory satisfying these conditions. Its preliminary formulations given by PODSTRIGACH and PAWLINA (1962) were based on the monograph by de GROOT and MAZUR (1951). This theory was fully and systematically developed by NOWACKI (1972). Lately some solutions of boundary problems have been obtained for elastic thermal diffusion (see [7], [12]). The most important experimental results and the references are given in [11], [12], [15], [16].

In this paper we shall deal with linear viscoelastic thermodiffusion. Thus, the memory of material will be taken into account both in mechanical and thermal–diffusive domains of the problems. Therefore constitutive equations derived for thermodynamic parts are derived in the way different from that applied to the elastic range.

The considerations are based on the system of five balances, determining the mutual interaction of thermal, diffusive and stress fields. This mass balance will be formulated only for the migrating component. The balances of momentum, angular momentum and the inequality expressing the increase of entropy have the classical form in which mass diffusion is not taken into account. The energy balance in which streams and sources have the components independent on mass, heat and momentum flows will have a different form. Hence, it follows that the interaction of mass, momentum and energy flows, occurring in thermodiffusion, will be included in the balance energy. The general expression of internal energy will be assumed as the functional depending on the whole history of the process, i.e., history of deformation, entropy and concentration. In linear problems we shall only approximate the functional of internal energy by linear and square functionals. This functional, being differentiated and introduced into the final inequality of process, after a suitable arrangement will give a set of physical equations for the process, determining the stress tensor, temperature and chemical potential as well as mass and heat streams. From the remaining components of the final inequality we obtain the dissipation function which can be used for investigations of properties of relaxation function centre. As the final result we obtain the conjugate set of thermodiffusion equations which, together with the set of initial–boundary conditions, allow us to determine the displacement field, entropy and diffusion field in the viscoelastic body.

The equations of viscoelastic thermodiffusion can be also formulated using the functional of free energy, obtained from internal energy as a result of Legendre transformation. In the functional the following histories will occur, namely those of the stress tensor, temperature and concentration. Transformations analogical to those in the case of internal energy yield the generating equations for the stress tensor, entropy and chemical potential. The generalized displacement equations and the generalized equation of thermodiffusion are the final result of such an approach.

In this paper, apart from the mentioned alternative approaches to viscoelastic thermodiffusion, an additional one will be analysed. It allows a deeper understanding of the structure of thermodiffusion equations in the viscoelastic body.

The presented approach to viscoelastic thermodiffusion is intended for analysis of thermodynamical foundations of the problems. The methods of solving the given boundary problems on the basis of the variational theorems are described by WYRWAŁ [16], [17].

2. THERMODIFFUSION BALANCES

Mass migration will be analysed with respect to the skeleton in a two-component body, the movement of the migrating component being referred to the crystal lattice of the stationary skeleton. Mass migration is caused by both the production of mass source r_2 and the substance supply from the outside due to the existence of the mass flux j_i. The mass flow is influenced by the stress field in the medium σ_{ij} and temperature Θ and the mentioned fields are also conjugated, like in the case of thermoviscoelasticity. The source and flux of energy will be completed. The source of energy will consist of 3 elements, i.e., heat element (ϱr_1 — the heat source), mechanical element ($\varrho F_i v_i$, ϱF_i being body force, v_i — particle velocity, and ϱ — medium density) and diffusive element (Mc, M is potential of the diffusing component and c, its concentration). Similarly, the energy flux will contain heat element (q_i — heat flux), mechanical element ($P_i v_i$, $P_i = \sigma_{ij} n_i$ being surface force, σ_{ij} — stress tensor and n_i vector of the normal) and diffusive element (Mj_i, j_i being the flux of mass flowing in time unit through an isolated, elementary surface plane). These fields are the functions of coordinates x_i, time t and will be referred to the initial body configuration.

The presented quantities will occur in balances of mass, momentum, angular momentum, energy and entropy, which in the case of viscoelastic thermodiffusion have the following forms:

1. Balance of diffusing mass

$$\frac{d}{dt} \int_V \varrho c \, dV = \int_V r_2 dV. \tag{2.1}$$

2. Balance of momentum

$$\frac{d}{dt}\int_V \varrho v_i dV = \int_V \varrho F_i dV + \int_A P_i dA. \tag{2.2}$$

3. Balance of angular momentum

$$\frac{d}{dt}\int_V \varrho \varepsilon_{ijk} x_j v_k dV = \int_V \varrho \varepsilon_{ijk} x_j F_k dV + \int_A \varepsilon_{ijk} x_j P_k dA. \tag{2.3}$$

4. Balance of energy

$$\frac{d}{dt}\int_V \varrho \left(U + \frac{1}{2} v_i v_i \right) dV = \int_V (\varrho r_1 + \varrho F_i v_i) dV + \int_A (P_i v_i - q_i n_i - Mj_i n_i) dA. \tag{2.4}$$

5. Inequality of entropy increase

$$\frac{d}{dt}\int_V \varrho S dV \geqslant \int_V \frac{\varrho r_1}{T} dV - \int_A \frac{1}{T} q_i n_i dA. \tag{2.5}$$

The local forms of balances (2.1)–(2.5) are the following:

$$\varrho \dot{c} = r_2 - j_{i,i}, \tag{2.1}'$$

$$\varrho \ddot{U}_i = \varrho F_i + \sigma_{ij,j}, \tag{2.2}'$$

$$\sigma_{ij} = \sigma_{ji}, \tag{2.3}'$$

$$\varrho \dot{U} = \varrho r_1 + \sigma_{ij} \dot{\varepsilon}_{ij} - q_{i,i} - (Mj_i)_{,i}, \tag{2.4}'$$

$$\varrho \dot{S} \geqslant \frac{\varrho r_1}{T} - \frac{q_{i,i}}{T} + \frac{q_i T_{,i}}{T^2}. \tag{2.5}'$$

In above equations ϱU, ϱS, T, ε_{ijk}, V, and A denote internal energy and entropy of the volume unit, temperature, Ricci permutation symbol, volume and surface of the medium, respectively, $(\cdot) \equiv \partial(\ )/\partial t$.

Further transformations of the last set of equations (2.1)'–(2.5)' yield the following inequality:

$$-\varrho \dot{U} + \varrho M \dot{c} + \sigma_{ij} \dot{\varepsilon}_{ij} + \varrho \dot{S} T - j_i M_{,i} - \frac{q_i T_{,i}}{T} \geqslant 0. \tag{2.6}$$

It should be satisfied for each real flow of heat, mass and momentum in a solid body, independently of the physical properties of the material.

Let us now introduce two new functions, i.e., increments of temperature $\Theta(x, t) = T(x, t) - T_0(x, t_0)$ and concentration $c(x, t) = c_1(x_1, t) - c_0(x, t_0)$ where T and c_1 denote the temperature and concentration at time t, and T_0 and c_0, their values in the reference configuration, characterized by the

absence of stresses in the medium. By introducing these quantities into the inequality (2.6) we obtain:

$$-\varrho\dot{U} + \varrho M\dot{c} + \sigma_{ij}\dot{\varepsilon}_{ij} + \varrho\dot{S}\Theta - j_i M_{,i} - \frac{q_i\Theta_{,i}}{T_0} \geq 0. \tag{2.7}$$

Inequality (2.7), completed with the constitutive assumptions determining the form of physical equations, especially that of internal energy, allows us to determine the first version of physical equations of thermodiffusion. The histories of deformation $\varepsilon_{ij}(s)$, concentration $c(s)$ and entropy $\varrho S(s)$ are the histories of the process Λ^1.

Let us introduce a new thermodynamical functional, i.e., free energy ϱA, determined by the relation:

$$\varrho A = \varrho U - \varrho\Theta S, \quad (-\varrho\dot{U} = -\varrho\dot{A} - \varrho\dot{\Theta}S - \varrho\Theta\dot{S}). \tag{2.8}$$

Introducing ϱA from (2.8) into the final inequality (2.7), we obtain:

$$-\varrho\dot{A} - \varrho S\dot{\Theta} + \varrho M\dot{c} + \sigma_{ij}\dot{\varepsilon}_{ij} - j_i M_{,i} - \frac{q_i\Theta_{,i}}{T_0} \geq 0. \tag{2.9}$$

This inequality, together with relation (2.8), was used for determination of thermodiffusion equations, presented in [11]. We shall also use another thermodynamical functional ϱK, determined by the relationship:

$$\varrho K = \varrho A - \varrho Mc, \quad (-\dot{A} = -\dot{K} - M\dot{c} - \dot{M}c). \tag{2.10}$$

The following final inequality is connected with the functional ϱK:

$$-\varrho\dot{K} - \varrho S\dot{\Theta} - \varrho c\dot{M} + \sigma_{ij}\dot{\varepsilon}_{ij} - j_i M_{,i} - \frac{q_i\Theta_{,i}}{T_0} \geq 0. \tag{2.11}$$

Obviously, in three different versions of thermodiffusion equations presented above, the histories of the process and generating functionals determining the problem are different. All these versions (there are six of them) are in principle equivalent, but their practical applications depend on the available experimental data, which favour the given version, e.g., that using free energy as the basic functional of the problem.

3. GENERATING EQUATIONS

In the previous item only a kind of the fields interaction was considered. Now we shall specify physical properties of the considered medium, which is described in so-called constructive assumptions.

To this end we give forms of those assumptions connected with the final inequalities (2.7), (2.9) and (2.11).

3.1. INTERNAL ENERGY ϱU

Histories of strain $\varepsilon_{ij}(s)$, entropy $\varrho S(s)$ and concentration $c(s)$ are connected with the functional

$$\varrho U = \varrho U(\Lambda^1) = \varrho \, \underset{s=0}{\overset{\infty}{\mathscr{U}}} \left(\varepsilon_{ij}(s), \varrho S(s), c(s) \right). \tag{3.1}$$

Thus, the problem is determined by the final inequality

$$-\varrho \dot{U} + \varrho M \dot{c} + \sigma_{ij} \dot{\varepsilon}_{ij} + \varrho \dot{S} \Theta - j_i M_{,i} - \frac{q_i \Theta_{,i}}{T_0} \geqslant 0 \tag{3.2}$$

and the generating equation

$$\varrho U = \varrho \, \underset{s=0}{\overset{\infty}{\mathscr{U}}} \left(\Lambda^1(t-s); \Lambda^1(t) \right),$$

$$\sigma_{ij} = \underset{s=0}{\overset{\infty}{\mathscr{P}}} \left(\Lambda^1(t-s); \Lambda^1(t) \right),$$

$$\Theta = \underset{s=0}{\overset{\infty}{\mathscr{2}}} \left(\Lambda^1(t-s); \Lambda^1(t) \right), \tag{3.3}$$

$$M = \underset{s=0}{\overset{\infty}{\mathscr{M}}} \left(\Lambda^1(t-s); \Lambda^1(t) \right),$$

$$j_i = \underset{s=0}{\overset{\infty}{\mathscr{J}}} (\text{grad } M), \qquad q_i = \underset{s=0}{\overset{\infty}{\mathscr{2}}} (\text{grad } \Theta).$$

In the case of linear problems, the functional ϱU should be approximated by the linear and square functional of the form:

$$\varrho U = \varrho U_0 + \int_{-\infty}^{t} D_{ij}(t-\tau) \dot{\varepsilon}_{ij}(\tau) d\tau + \int_{-\infty}^{t} \beta^1(t-\tau) \dot{S}(\tau) d\tau$$

$$+ \int_{-\infty}^{t} \pi(t-\tau) \dot{c}(\tau) d\tau$$

$$+ \frac{1}{2} \int\!\!\int_{-\infty}^{tt} E_{ijkl}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{\varepsilon}_{kl}(\tau') d\tau d\tau'$$

$$+ \int\!\!\int_{-\infty}^{tt} \varphi_{ij}^1(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{S}(\tau') d\tau d\tau'$$

$$+ \frac{1}{2} \int\!\!\int_{-\infty}^{tt} m^1(t-\tau, t-\tau') \dot{S}(\tau) \dot{S}(\tau') d\tau d\tau'$$

$$+ \iint_{-\infty}^{tt} l^1(t-\tau, t-\tau') \dot{c}(\tau) \dot{S}(\tau') d\tau d\tau'$$

$$+ \frac{1}{2} \iint_{-\infty}^{tt} n(t-\tau, t-\tau') \dot{c}(\tau) \dot{c}(\tau') d\tau d\tau'$$

$$+ \iint_{-\infty}^{tt} \Phi_{ij}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{c}(\tau') d\tau d\tau' + \mathcal{O}(\varepsilon^3) \tag{3.4}$$

where ϱU_0 is the value of internal energy in the reference configuration. The relaxation functions D_{ij}, β^1, π, E_{ijkl}, φ_{ij}^1, m^1, n, l^1 and Φ_{ij} are continuous for $\tau \geqslant 0$, $\tau' \geqslant 0$ and disappear for $\tau < 0$ and $\tau' < 0$. Internal energy ϱU after its differentiation is put in (3.2) from which, after a suitable arrangement, we obtain the following set of generating equations for:

1. Stress tensor

$$\sigma_{ij}(t) = D_{ij}(0) + \int_{-\infty}^{t} E_{ijkl}(t-\tau, 0) \dot{\varepsilon}_{kl}(\tau) d\tau$$

$$+ \int_{-\infty}^{t} \varphi_{ij}^1(0, t-\tau) \dot{S}(\tau) d\tau + \int_{-\infty}^{t} \Phi_{ij}(0, t-\tau) \dot{c}(\tau) d\tau. \tag{3.5}$$

2. Temperature

$$\Theta(t) = \beta^1(0) + \int_{-\infty}^{t} \varphi_{ij}(t-\tau, 0) \dot{\varepsilon}_{ij}(\tau) d\tau$$

$$+ \int_{-\infty}^{t} m^1(t-\tau, 0) \dot{S}(\tau) d\tau + \int_{-\infty}^{t} l^1(t-\tau, 0) \dot{c}(\tau) d\tau. \tag{3.6}$$

3. Chemical potential

$$M(t) = \pi(0) + \int_{-\infty}^{t} \Phi_{ij}(t-\tau, 0) \dot{\varepsilon}_{ij}(\tau) d\tau$$

$$+ \int_{-\infty}^{t} n(0, t-\tau) \dot{c}(\tau) d\tau + \int_{-\infty}^{t} l^1(0, t-\tau) \dot{S}(\tau) d\tau. \tag{3.7}$$

4. The inequality from the remaining components of (3.2)

$$\int_{-\infty}^{t} \frac{\partial}{\partial t} D_{ij}(t-\tau) \dot{\varepsilon}_{ij}(\tau) d\tau + \int_{-\infty}^{t} \frac{\partial}{\partial t} \beta^1(t-\tau) \dot{S}(\tau) d\tau$$

$$+ \int_{-\infty}^{t} \frac{\partial}{\partial t} \pi(t-\tau) \dot{c}(\tau) d\tau + \Omega^1 - j_i M_{,i} - \frac{q_i \Theta_{,i}}{(T_0)^2} \geqslant 0. \tag{3.8}$$

In this case, the dissipation function Ω_1 has the form:

$$\Omega_1 = \frac{1}{2} \int\int\limits_{-\infty}^{tt} \frac{\partial}{\partial t} E_{ijkl}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{\varepsilon}_{kl}(\tau') \, d\tau d\tau'$$

$$+ \int\limits_{-\infty}^{tt} \frac{\partial}{\partial t} \varphi_{ij}^1(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{S}(\tau') \, d\tau d\tau'$$

$$+ \frac{1}{2} \int\int\limits_{-\infty}^{tt} \frac{\partial}{\partial t} m^1(t-\tau, t-\tau') \dot{S}(\tau) \dot{S}(\tau') \, d\tau d\tau'$$

$$+ \int\int\limits_{-\infty}^{tt} \frac{\partial}{\partial t} \Phi_{ij}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{c}(\tau') \, d\tau d\tau'$$

$$+ \frac{1}{2} \int\int\limits_{-\infty}^{tt} \frac{\partial}{\partial t} n(t-\tau, t-\tau') \dot{c}(\tau) \dot{c}(\tau') \, d\tau d\tau'$$

$$+ \int\int\limits_{-\infty}^{tt} \frac{\partial}{\partial t} l^1(t-\tau, t-\tau') \dot{c}(\tau) \dot{S}(\tau') \, d\tau d\tau'. \tag{3.9}$$

From the postulate that the inequality (3.8) be valid for each thermodiffusion process, we get immediately:

$$\frac{\partial}{\partial t} D_{ij}(t-\tau) = 0, \qquad \frac{\partial}{\partial t} \beta^1(t-\tau) = 0, \qquad \frac{\partial}{\partial t} \pi(t-\tau) = 0,$$

whereas for the homogeneous thermal field ($\Theta_{,i} = 0$) and chemical potential ($M_{,i} = 0$) we have:

$$\Omega^1 > 0. \tag{3.10}$$

Thus, for both streams we obtain the following constraints

$$-j_i M_{,i} \geq 0 \quad \text{and} \quad -q_i \Theta_{,i} \geq 0 \tag{3.11}$$

from which it results that constitutive equations for the streams can have only the following form:

$$j_i = -k_{ij} M_{,j}, \qquad q_i = -K_{ij}\Theta_{,j} \tag{3.12}$$

that corresponds to classical Fick–Fourier equations.

Thus, we have obtained a set of physical equations determining the thermodiffusion process.

Taking account of eqs. (3.4)–(3.7) and (3.12) in the energy balance (2.4) and neglecting non-linear terms Ω^1, we obtain:

$$\varrho r_1 - T_0 \varrho \dot{S} + (K_{ij}\Theta_{,j})_{,i} = 0, \tag{3.13}$$

$$\varrho r_1 - T_0 \varrho \dot{S} + [K_{ln}(\int\limits_{-\infty}^{t} \varphi_{ij}^1(t-\tau,0)\dot{\varepsilon}_{ij}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} m^1(t-\tau,0)\dot{S}(\tau)\,d\tau + \int\limits_{-\infty}^{t} l^1(t-\tau,0)\dot{c}(\tau)\,d\tau)]_{,nl} = 0. \quad (3.14)$$

Analogically, we shall obtain equations of diffusion:

$$r_2 - \varrho \dot{c} - (k_{ij}M_{,j})_{,i} = 0, \quad (3.15)$$

$$r_2 - \varrho \dot{c} - [k_{ln}(\int\limits_{-\infty}^{t} n(0,t-\tau)\dot{c}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} l^1(0,t-\tau)\dot{S}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \Phi_{ij}(t-\tau,0)\dot{\varepsilon}_{ij}(\tau)\,d\tau)]_{,nl} = 0. \quad (3.16)$$

Isotropic cases of equation of heat and mass flow lead, however, to the relationships:

$$\varrho r_1 - T_0 \dot{S} + K[\int\limits_{-\infty}^{t} \varphi^1(t-\tau)\dot{\varepsilon}_{jj}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} m^1(t-\tau)\dot{S}(\tau)\,d\tau + \int\limits_{-\infty}^{t} l^1(t-\tau)\dot{c}(\tau)\,d\tau]_{,ii} = 0,$$

$$(\varrho r_1 - T_0 \dot{S} + K[\varphi^1 * \dot{\varepsilon}_{jj} + m^1 * \dot{S} + l^1 * \dot{c}]_{,ii} = 0), \quad (3.17)$$

$$r_2 - \varrho \dot{c} - k[\int\limits_{-\infty}^{t} n(t-\tau)\dot{c}(\tau)\,d\tau + \int\limits_{-\infty}^{t} l^1(t-\tau)\dot{S}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} \Phi(t-\tau)\dot{\varepsilon}_{ii}(\tau)\,d\tau]_{,ii} = 0, \quad (3.18)$$

$$(r_2 - \dot{c} - [kn * \dot{c} + l^1 * \dot{S} + \Phi * \dot{\varepsilon}_{ll}]_{,ii} = 0)$$

in which we have assumed:

$$\varphi_{ij}^1(t-\tau,0) = \varphi^1(t-\tau)\delta_{ij}, \qquad \Phi_{ij}(t-\tau,0) = \Phi(t-\tau)\delta_{ij},$$

$$K_{ij} = K\delta_{ij}, \qquad k_{ij} = k\delta_{ij}, \quad (3.19)$$

$$f_1 * f_2 = \int\limits_{-\infty}^{t} f_1(t-\tau) f_2(\tau)\,d\tau.$$

Let us notice that the operators of heat and mass flow equations have the same forms.

3.2. FREE ENERGY ϱA

When the formulation of thermodiffusion problems based on the notion of free energy, then histories of strain $\varepsilon_{ij}(s)$, temperature $\Theta(s)$ and concentration $c(s)$ will occur in the process as the independent histories. The history vector has thus the following form:

$$\varLambda^{2T} = [\varepsilon_{ij}(s), \ \Theta(s), c(s)]. \tag{3.20}$$

Free energy, which is here a basic functional and is determined as follows:

$$\varrho A = \varrho \mathop{\mathscr{A}}_{s=0}^{\infty} \left(\varLambda^2(t-s); \ \varLambda^2(t)\right), \tag{3.21}$$

is connected with internal energy by the relation:

$$\varrho A = \varrho \mathop{\mathscr{U}}_{s=0}^{\infty} \left(\varLambda^1(t-s); \varLambda^1(t)\right) - \varrho s \mathop{\mathscr{Q}}_{s=0}^{\infty} \left(\varLambda^1(t-s); \varLambda^1(t)\right)$$

$$= \varrho \mathop{\mathscr{A}}_{s=0}^{\infty} \left(\varLambda^2(t-s); \varLambda^2(t)\right). \tag{3.22}$$

In this case the final inequality and the generating equations have the following forms:

$$-\varrho\dot{A} - \varrho S\dot{\Theta} + \varrho M\dot{c} + \sigma_{ij}\dot{\varepsilon}_{ij} - j_i M_{,i} - \frac{q_i \Theta_{,i}}{(T_0)^2} \geqslant 0, \tag{3.23}$$

$$\varrho A = \varrho \mathop{\mathscr{A}}_{s=0}^{\infty} \left(\varLambda^2(t-s); \varLambda^2(t)\right),$$

$$\sigma_{ij} = \mathop{\mathscr{P}}_{s=0}^{\infty} \left(\varLambda^2(t-s); \varLambda^2(t)\right), \qquad -\varrho S = \mathop{\mathscr{I}}_{s=0}^{\infty} \left(\varLambda^2(t-s); \varLambda^2(t)\right), \tag{3.24}$$

$$M = \mathop{\mathscr{M}}_{s=0}^{\infty} \left(\varLambda^2(t-s); \varLambda^2(t)\right), \qquad j_i = \mathop{\mathscr{J}}_{s=0}^{\infty} (\mathrm{grad}\, M), \qquad q_i = \mathop{\mathscr{Q}}_{s=0}^{\infty} (\mathrm{grad}\, \Theta).$$

Confining ourselves to the linear problems, we approximate the functional of free energy only by the linear and square functionals to get:

$$\varrho A = \varrho A_0 + \int\limits_{-\infty}^{t} D_{ij}(t-\tau)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} \beta^2(t-\tau)\,\dot{\Theta}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \pi(t-\tau)\,\dot{c}(\tau)\,d\tau$$

$$+ \frac{1}{2} \iint\limits_{-\infty}^{tt} E_{ijkl}(t-\tau, t-\tau')\dot{\varepsilon}_{ij}(\tau)\,\dot{\varepsilon}_{kl}(\tau')\,d\tau d\tau'$$

$$+ \iint\limits_{-\infty}^{tt} \varphi_{ij}^2(t-\tau, t-\tau')\,\dot{\varepsilon}_{ij}(\tau)\,\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \iint\limits_{-\infty}^{tt} l^2(t-\tau, t-\tau')\,\dot{c}(\tau)\,\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \frac{1}{2} \iint\limits_{-\infty}^{tt} m^2(t-\tau, t-\tau')\,\dot{\Theta}(\tau)\,\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \frac{1}{2} \iint\limits_{-\infty}^{tt} n(t-\tau, t-\tau')\,\dot{c}(\tau)\,\dot{c}(\tau')\,d\tau d\tau'$$

$$+ \iint\limits_{-\infty}^{tt} \Phi_{ij}(t-\tau, t-\tau')\,\dot{\varepsilon}_{ij}(\tau)\,\dot{c}(\tau')\,d\tau d\tau' + o(\varepsilon^3). \tag{3.25}$$

Like previously, ϱA_0 is free energy in the reference configuration, D_{ij}, β^2, π, E_{ijkl}, φ_{ij}^2, m^2, n^2, l, Φ_{ij} are the relaxation functions, determining physical properties of the material. The functional (3.25) being differentiated is put to inequality (3.23) from which, after a suitable arrangement, we obtain the following generating equations for:

1. Stress tensor

$$\sigma_{ij}(t) = D_{ij}(0) + \int\limits_{-\infty}^{t} E_{ijkl}(t-\tau, 0)\,\dot{\varepsilon}_{kl}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} \varphi_{ij}^2(0, t-\tau)\,\dot{\Theta}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \Phi_{ij}(0, t-\tau)\,\dot{c}(\tau)\,d\tau. \tag{3.26}$$

2. Entropy

$$-\varrho S(t) = \beta^2(0) + \int\limits_{-\infty}^{t} \varphi_{ij}^2(t-\tau, 0)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} m^2(t-\tau, 0)\,\dot{\Theta}(\tau)\,d\tau + \int\limits_{-\infty}^{t} l^2(t-\tau, 0)\,\dot{c}(\tau)\,d\tau. \tag{3.27}$$

3. Chemical potential

$$M(t) = \pi(0) + \int\limits_{-\infty}^{t} n(0, t-\tau)\,\dot{c}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} l^2(0, t-\tau)\,\dot{\Theta}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \Phi_{ij}(t-\tau, 0)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau. \tag{3.28}$$

From the remaining components we obtain the following inequality:

$$\int\limits_{-\infty}^{t} \frac{\partial}{\partial t} D_{ij}(t-\tau)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \frac{\partial}{\partial t} \beta^2(t-\tau)\,\dot{\Theta}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} \frac{\partial}{\partial t} \pi(t-\tau)\,\dot{c}(\tau)\,d\tau + \Omega^2 - j_i M_{,i} - \frac{q_i \Theta_{,i}}{(T_0)^2} \geq 0. \tag{3.29}$$

The dissipation function Ω^2, occurring in inequality (3.29), is determined by the relation:

$$\Omega^2 = \frac{1}{2} \int\int_{-\infty}^{tt} \frac{\partial}{\partial t} E_{ijkl}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{\varepsilon}_{kl}(\tau') \, d\tau d\tau'$$

$$+ \int\int_{-\infty}^{tt} \frac{\partial}{\partial t} \varphi_{ij}^2(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{\Theta}(\tau') \, d\tau d\tau'$$

$$+ \frac{1}{2} \int\int_{-\infty}^{tt} \frac{\partial}{\partial t} m^2(t-\tau, t-\tau') \dot{\Theta}(\tau) \dot{\Theta}(\tau') \, d\tau d\tau'$$

$$+ \int\int_{-\infty}^{tt} \frac{\partial}{\partial t} \Phi_{ij}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{c}(\tau') \, d\tau d\tau'$$

$$+ \frac{1}{2} \int\int_{-\infty}^{tt} \frac{\partial}{\partial t} n(t-\tau, t-\tau') \dot{c}(\tau) \dot{c}(\tau') \, d\tau d\tau'$$

$$+ \int\int_{-\infty}^{tt} \frac{\partial}{\partial t} l^2(t-\tau, t-\tau') \dot{c}(\tau) \dot{\Theta}(\tau') \, d\tau d\tau'. \tag{3.30}$$

Like in the previous case, from the analysis of the inequality (3.29) we obtain limiting conditions for the linear part of the functional

$$\frac{\partial}{\partial t} D_{ij}(t-\tau) = 0, \quad \frac{\partial}{\partial t} \beta^2(t-\tau) = 0, \quad \frac{\partial}{\partial t} \pi(t-\tau) = 0$$

and the relation

$$\Omega^2 > 0 \tag{3.31}$$

which is the basic constraint for the dissipation function Ω^2.

The form of physical equations determining both streams is not changed, i.e.,

$$j_i = -k_{ij} M_{,j}, \quad q_i = -K_{ij}\Theta_{,j}. \tag{3.32}$$

The conjugate equations of mass and heat flows are obtained by the same procedure. In this case, however, the applied physical equation does not determine temperature (like in the former case) but entropy. The transformation yields a set of equations

$$\varrho r_1 + T_0 \varrho \frac{\partial}{\partial t} \Big[\int_{-\infty}^{t} \varphi_{ij}^2(t-\tau, 0) \dot{\varepsilon}_{ij}(\tau) \, d\tau + \int_{-\infty}^{t} m^2(t-\tau, 0) \dot{\Theta}(\tau) \, d\tau$$

$$+ \int_{-\infty}^{t} l^2(t-\tau, 0) \dot{c}(\tau) \, d\tau \Big] + (K_{ij}\Theta_{,j})_{,i} = 0, \tag{3.33}$$

$$r_2 - \varrho\dot{c} - k_{ln}\Big[\int_{-\infty}^{t} n(0, t-\tau)\varrho\dot{c}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} l^2(0, t-\tau)\dot{\Theta}(\tau)\,d\tau + \int_{-\infty}^{t} \Phi_{ij}(t-\tau, 0)\dot{\varepsilon}_{ij}(\tau)\,d\tau\Big]_{,nl} = 0 \qquad (3.34)$$

for the anisotropic material and

$$\varrho r_1 - T_0\varrho\frac{\partial}{\partial t}\Big[\int_{-\infty}^{t} \varphi^2(t-\tau)\dot{\varepsilon}_{jj}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} m^2(t-\tau)\dot{\Theta}(\tau)\,d\tau + \int_{-\infty}^{t} l^2(t-\tau)\dot{c}(\tau)\,d\tau + K\Theta_{,ii} = 0,$$
$$\qquad (3.35)$$

$$r_2 - \varrho\dot{c} - K\Big[\int_{-\infty}^{t} n(t-\tau)\dot{c}(\tau)\,d\tau + \int_{-\infty}^{t} l^2(t-\tau)\dot{\Theta}(\tau)\,d\tau + \int_{-\infty}^{t} \Phi(t-\tau)\dot{\varepsilon}_{jj}(\tau)\,d\tau\Big]_{,ii}$$
$$= 0$$

for the isotropic material.

In the latter case the relaxation functions have the forms:

$$\varphi_{ij}^2(t-\tau, 0) = \varphi^2(t-\tau)\delta_{ij}, \qquad \Phi_{ij}(t-\tau, 0) = \Phi(t-\tau)\delta_{ij},$$
$$K_{ij} = K\delta_{ij}, \qquad k_{ij} = k\delta_{ij}. \qquad (3.36)$$

In both equations the operators are different, contrary to the previous case.

The set of coupled thermodiffusion equations is obtained after substituting the equation determining the stress tensor (3.34) or (3.35) to the momentum equations. In the case of isotropy we obtain the following set of equations:

$$\mu * du_{i,jj} + (\lambda + \mu) * du_{j,ji} + \varrho F_i - \gamma_T * d\Theta_{,i} - \gamma_c * dC_{,i} = \varrho\ddot{u}_i,$$

$$\varrho r_2 - T_0\frac{\partial}{\partial t}[\gamma_T * du_{i,i} + m^2 * d\Theta + l^2 * dC] + k\Theta_{,ii} = 0, \qquad (3.37)$$

$$r_1 - \varrho\dot{c} - k[n * dC + l^2 * d\Theta + \gamma_c * du_{i,i}]_{,jj} = 0$$

which, after completion with the initial and boundary conditions, determines the boundary problem of viscoelastic thermodiffusion. The boundary conditions of heat and mechanical parts have the classical form, whereas in the diffusive part we most often use chemical potential and its gradient that determines the mass exchange with the surroundings.

Symbol $f_1 * df_2$ in equations (3.37) denotes Stieltjes convolution, whereas the physical equation determining the stress tensor in the isotropic body has the form $\sigma_{ij} = 2\mu * d\varepsilon_{ij} + \lambda * d\varepsilon_{kk}\delta_{ij} - \gamma_c * dc\delta_{ij} - \gamma_T * d\Theta\delta_{ij}$ where μ, λ, $\gamma_c = \alpha_c(3\lambda + 2\mu)$, $\gamma_T = \alpha_T(3\lambda + 2\mu)$ are the relaxation functionals.

The next form of thermodiffusion equations is obtained by using as the basic functional the functional ϱK, defined by the relation $\varrho K = \varrho A + \varrho M c$, connected with free energy ϱA. Then the process is determined by histories of strain $\varepsilon_{ij}(s)$, temperature $\Theta(s)$ and chemical potential $M(s)$. The vector of history Λ^3 in the space of history of the process has the form:

$$\Lambda^{3T} = [\varepsilon_{ij}(s), \Theta(s), M(s)] \tag{3.38}$$

and the functional ϱK is determined by the relation:

$$\varrho K = \varrho \underset{s=0}{\overset{\infty}{\mathscr{K}}} \left(\Lambda^3(t-s); \Lambda^3(t) \right)$$

$$= \varrho \underset{s=0}{\overset{\infty}{\mathscr{A}}} \left(\Lambda^2(t-s); \Lambda^2(t) \right) - \varrho c \underset{s=0}{\overset{\infty}{\mathscr{M}}} \left(\Lambda^2(t-s); \Lambda^2(t) \right). \tag{3.39}$$

In such an approach mass and heat exchanges are determined by: the final inequality

$$-\varrho \dot{K} - \varrho S \dot{\Theta} - \varrho c \dot{M} + \sigma_{ij} \dot{\varepsilon}_{ij} - j_i M_{,i} - \frac{q_i \Theta_{,i}}{(T_0)^2} \geqslant 0 \tag{3.40}$$

and the generating equations

$$\varrho K = \varrho \underset{s=0}{\overset{\infty}{\mathscr{K}}} \left(\Lambda^3(t-s); \Lambda^3(t) \right),$$

$$\sigma_{ij} = \underset{s=0}{\overset{\infty}{\mathscr{P}}} \left(\Lambda^3(t-s); \Lambda^3(t) \right),$$

$$-\varrho S = \underset{s=0}{\overset{\infty}{\mathscr{I}}} \left(\Lambda^3(t-s); \Lambda^3(t) \right), \qquad -c = \underset{s=0}{\overset{\infty}{\mathscr{C}}} \left(\Lambda^3(t-s); \Lambda^3(t) \right), \tag{3.41}$$

$$q_i = \underset{s=0}{\overset{\infty}{\mathscr{D}}} (\operatorname{grad} \Theta), \qquad j_i = \underset{s=0}{\overset{\infty}{\mathscr{J}}} (\operatorname{grad} M).$$

Taking the previous assumptions concerning the functional ϱK, i.e., limiting ourselves to its approximation by the square functional, we obtain a new set of linear physical equations of viscoelastic thermodiffusion:

$$\varrho K = \varrho K_0 + \int_{-\infty}^{t} D_{ij}(t-\tau) \dot{\varepsilon}_{ij}(\tau) \, d\tau + \int_{-\infty}^{t} \beta^2(t-\tau) \dot{\Theta}(\tau) \, d\tau$$

$$+ \int_{-\infty}^{t} \pi^3(t-\tau) \dot{M}(\tau) \, d\tau$$

$$+ \frac{1}{2} \underset{-\infty}{\overset{tt}{\iint}} E_{ijkl}(t-\tau, t-\tau') \dot{\varepsilon}_{ij}(\tau) \dot{\varepsilon}_{kl}(\tau') \, d\tau d\tau'$$

$$+ \int\limits_{-\infty}^{tt} \varphi_{ij}^2(t-\tau,t-\tau')\,\dot{\varepsilon}_{ij}(\tau)\,\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \frac{1}{2} \iint\limits_{-\infty}^{tt} m^2(t-\tau,t-\tau')\,\dot{\Theta}(\tau)\,\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \frac{1}{2} \iint\limits_{-\infty}^{tt} n^3(t-\tau,t-\tau')\,\dot{M}(\tau)\,\dot{M}(\tau')\,d\tau d\tau'$$

$$+ \iint\limits_{-\infty}^{tt} l^3(t-\tau,t-\tau')\,\dot{M}(\tau)\,\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \iint\limits_{-\infty}^{tt} \Phi_{ij}^3(t-\tau,t-\tau')\,\dot{\varepsilon}_{ij}(\tau)\,\dot{M}(\tau')\,d\tau d\tau' + \mathcal{O}(\varepsilon^3). \tag{3.42}$$

Expression (3.42) is the sought for approximation of the functional ϱK by the linear and square functional. ϱK_0 denotes the initial value of the functional ϱK in the reference configuration. D_{ij}, β^2, π^3, E_{ijkl}, φ_{ij}^2, Φ_{ij}^3, m^2, n^3, l^3 are the successive sets of relaxation functions the values of which should be determined experimentally.

The set of generating equations is obtained from the inequality (3.40) after substituting the derivative ϱK of the functional (3.42) and a suitable arrangement of the obtained inequality. As a result we obtain the following set of physical equations for:

stress tensor

$$\sigma_{ij}(t) = D_{ij}(0) + \int\limits_{-\infty}^{t} E_{ijkl}(t-\tau,0)\,\dot{\varepsilon}_{kl}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} \varphi_{ij}^2(0,t-\tau)\,\dot{\Theta}(\tau)\,d\tau + \int\limits_{-\infty}^{t} \Phi_{ij}^3(0,t-\tau)\,\dot{M}(\tau)\,d\tau, \tag{3.43}$$

entropy

$$-\varrho S(t) = \beta^2(0) + \int\limits_{-\infty}^{t} \varphi_{ij}^2(t-\tau,0)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} m^2(t-\tau,0)\,\dot{\Theta}(\tau)\,d\tau + \int\limits_{-\infty}^{t} l^3(t-\tau,0)\,\dot{M}(\tau)\,d\tau, \tag{3.44}$$

concentration

$$-\varrho c(t) = \pi^3(0) + \int\limits_{-\infty}^{t} \Phi_{ij}^3(t-\tau,0)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau$$

$$+ \int\limits_{-\infty}^{t} n^3(0,t-\tau)\,\dot{M}(\tau)\,d\tau + \int\limits_{-\infty}^{t} l^3(0,t-\tau)\,\dot{\Theta}(\tau)\,d\tau, \tag{3.45}$$

and from the remaining component of the process the inequality

$$\int_{-\infty}^{t} \frac{\partial}{\partial t} D_{ij}(t-\tau)\dot{\varepsilon}_{ij}(\tau)\,d\tau + \int_{-\infty}^{t} \frac{\partial}{\partial t} \beta^2(t-\tau)\dot{\Theta}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} \frac{\partial}{\partial t} \pi^3(t-\tau)\dot{M}(\tau)\,d\tau + \Omega^3 - j_i M_{,i} - \frac{q_i \Theta_{,i}}{(T_0)^2} \geqslant 0. \quad (3.46)$$

At the same time the new dissipation function Ω^3 will take the following form

$$\Omega^3 = \frac{1}{2} \int\!\!\int_{-\infty}^{tt} \frac{\partial}{\partial t} E_{ijkl}(t-\tau, t-\tau')\dot{\varepsilon}_{ij}(\tau)\dot{\varepsilon}_{kl}(\tau')\,d\tau d\tau'$$

$$+ \int\!\!\int_{-\infty}^{tt} \frac{\partial}{\partial t} \varphi_{ij}^2(t-\tau, t-\tau')\dot{\varepsilon}_{ij}(\tau)\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \int\!\!\int_{-\infty}^{tt} \frac{\partial}{\partial t} \Phi_{ij}^3(t-\tau, t-\tau')\dot{\varepsilon}_{ij}(\tau)\dot{M}(\tau')\,d\tau d\tau'$$

$$+ \frac{1}{2} \int\!\!\int_{-\infty}^{tt} \frac{\partial}{\partial t} m^2(t-\tau, t-\tau')\dot{\Theta}(\tau)\dot{\Theta}(\tau')\,d\tau d\tau'$$

$$+ \frac{1}{2} \int\!\!\int_{-\infty}^{tt} \frac{\partial}{\partial t} n^3(t-\tau, t-\tau')\dot{M}(\tau)\dot{M}(\tau')\,d\tau d\tau'$$

$$+ \int\!\!\int_{-\infty}^{tt} \frac{\partial}{\partial t} l^3(t-\tau, t-\tau')\dot{M}(\tau)\dot{\Theta}(\tau')\,d\tau d\tau'. \quad (3.47)$$

By the same argumentation as that in the previous section we obtain the constraints for:

relaxation function

$$\frac{\partial}{\partial t} D_{ij}(t-\tau) = 0, \qquad \frac{\partial}{\partial t} \beta^2(t-\tau) = 0, \qquad \frac{\partial}{\partial t} \pi^3(t-\tau) = 0,$$

expression determining the dissipation:

$$\Omega^3 > 0. \quad (3.48)$$

Finally we obtain the equations for the fluxes of mass and heat

$$j_i = -k_{ij}M_{,j}, \qquad q_i = -K_{ij}\Theta_{,j}. \quad (3.49)$$

The presented set of physical equations for viscoelastic thermodiffusion allows us to derive the coupled equations for the process considered. They

are obtained by substituting the generating equations into balances of mass, momentum and energy and by linearization of the conductivity equation. The heat and diffusion equations have the following forms for:

anisotropic body

$$\varrho r_1 + T_0\varrho \frac{\partial}{\partial t}\Big[\int_{-\infty}^{t} \varphi_{ij}^2(t-\tau,0)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau + \int_{-\infty}^{t} m^2(t-\tau,0)\,\dot{\Theta}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} l^3(t-\tau,0)\,\dot{M}(\tau)\,d\tau\Big] + (K_{ij}\Theta_{,j})_{,i} = 0,$$

$$r_2 + \frac{\partial}{\partial t}\Big[\int_{-\infty}^{t} n^3(t-\tau,0)\,\dot{M}(\tau)\,d\tau + \int_{-\infty}^{t} l^3(0,t-\tau)\,\dot{\Theta}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} \Phi_{ij}^3(t-\tau,0)\,\dot{\varepsilon}_{ij}(\tau)\,d\tau\Big] + (k_{ij}M_{,j})_{,i} = 0, \qquad (3.50)$$

isotropic material:

$$\varrho r_1 + T_0\varrho \frac{\partial}{\partial t}\Big[\int_{-\infty}^{t} \varphi^2(t-\tau)\,\dot{\varepsilon}_{ii}(\tau)\,d\tau + \int_{-\infty}^{t} m^2(t-\tau)\,\dot{\Theta}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} l^3(t-\tau)\,\dot{M}(\tau)\,d\tau\Big] + K\Theta_{,jj} = 0,$$

$$r_2 + \frac{\partial}{\partial t}\Big[\int_{-\infty}^{t} n^3(t-\tau)\,\dot{M}(\tau)\,d\tau + \int_{-\infty}^{t} l^3(t-\tau)\,\dot{\Theta}(\tau)\,d\tau$$

$$+ \int_{-\infty}^{t} \Phi^3(t-\tau)\,\dot{\varepsilon}_{ii}(\tau)\,d\tau\Big] + kM_{,jj} = 0 \qquad (3.51)$$

and

$$\varrho r_1 + T_0\frac{\partial}{\partial t}\big[\varphi^2 * \dot{\varepsilon}_{ii} + m^2 * \dot{\Theta} + l^3 * \dot{M}\big] + K\Theta_{,jj} = 0,$$

$$r_2 + \frac{\partial}{\partial t}\big[n^3 * \dot{M} + l^3 * \dot{\Theta} + \Phi^3 * \dot{\varepsilon}_{ii}\big] + kM_{,jj} = 0.$$

Like in the case of the first coupled equations (3.14) and (3.16), we see that the operators of the equations of diffusion and heat flow are similar. Similarity of these operators determines the essential physical property of the conjugate equations of thermodiffusion.

For the mechanical part the equation is obtained by substituting the equations determining the stress tensor into the equations of momentum balance. The set of the equations of linear viscoelastic thermodiffusion has

then the form:

$$\mu * du_{i,jj} + (\lambda + \mu) * du_{j,ji} + \varrho F_i - \gamma_T * d\Theta_{,i} - \gamma_c * dM_{,i} - \varrho\ddot{u}_i = 0,$$

$$\varrho r_1 + \varrho T_0 \frac{\partial}{\partial t}[\varphi_2 * d\varepsilon_{ii} + m^2 * d\Theta + l^3 * dM] + K\Theta_{,ii} = 0, \qquad (3.52)$$

$$r_2 + \frac{\partial}{\partial t}[n_3 * dM + l^3 * d\Theta + \Phi^3 * d\varepsilon_{ii}] + kM_{,ii} = 0.$$

It is a set of five differential–integral equations, where three components of the displacement vector, temperature and chemical potential are the searched quantities.

By introducing the initial and boundary conditions into the set of equations (3.52), we can formulate the full initial–boundary problems. The simplest problems are determined by the displacement, temperature and chemical potential fields on the respective parts of the end.

The procedure presented in this section, which gave three equivalent approaches to the boundary problems of thermodiffussion, can be developed using new thermodynamic functionals determined on the history of stress tensor. These problems will not be studied since they do not introduce any new ideas.

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